HEAT TRANSFER IN APPARATUS WITH A MIXER

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This paper gives the results of an approximate theoretical analysis of the problem of heat transfer between the wall of the apparatus with a mixer. The design equations obtained are compared with the experimental data of various investigators.

The results of the main investigations of heat transfer in apparatus with mixers have been summed up in [1]. Despite their great practical interest, the number of investigations on this question is inadequate in view of the extensive use of such apparatus.

The lack of theoretical generalizations prevents the available design equations being extended to mixers which differ slightly from the investigated types.

This paper presents an approximate theoretical analysis of the problem of heat transfer in apparatus with a mixer in the case of turbulent mixing. An exact solution of this problem is very difficult, since the structure of the flow in an apparatus with a mixer is complex and depends on the type of mixer.

An analysis of the experimental data on heat transfer and power consumption, however, indicates local similarity of the fields of turbulent pulsations for different types of mixers, despite their different design. The existence of an approximate similarity between the fields of turbulent pulsations allows a common approach to the problem of heat transfer in apparatus with mixers of different designs and other similar cases.

Heat transfer to the wall of the apparatus. In turbulent conditions heat transfer is effected by the turbulent vortices of the liquid which flow over the heat-transfer surface. The motion of the liquid in apparatus with a mixer is three-dimensional in nature. In a first approximation we can assume that turbulent pulsating movements in any direction relative to the surface are equally probable, i.e., the turbulence is isotropic.

This means that at any instant one-third of the surface is washed by a stream perpendicular to the wall, and two-thirds of the surface is washed by a stream of liquid parallel to the surface.

Assuming that pulsations of different direction participate to the same extent in heat transfer, we can express the mean heat transfer coefficient in the following way:

$$\alpha = \frac{1}{3}\alpha_n + \frac{2}{3}\alpha_t.$$

For a surface washed by a stream perpendicular to it we know from theoretical and experimental work [2] that

$$\mathrm{Nu}_n = \alpha_n d/\lambda = 1.15\ \mathrm{Re}_n^{0.5}\,\mathrm{Pr}^{1/3}(\mu/\mu_w)^{0.14}. \quad (1)$$

For the initial section of a surface washed by a parallel turbulent stream of liquid

$$\mathrm{Nu}_t = \alpha_t d/\lambda = 0.035\,\mathrm{Re}_t^{0.8}\,\mathrm{Pr}^{1/3}(\mu/\mu_w)^{0.14}. \quad (2)$$

Thus, the problem of determining the heat transfer coefficient reduces to the determination of the characteristic dimension l and the mean fluctuation velocity u_p.

Following Levich, we can take

$$l = d_м/2.$$

We now consider the energy balance in an apparatus with a mixer. The power expended on mixing the liquid is

$$N = \xi_м \rho\, n^3 d_м^5. \quad (3)$$

This power is equal to the kinetic power of all the streams:

$$N = \Sigma N_i\, i. \quad (4)$$

For a single stream we can write

$$N_i = m_s u_p^2/2.$$

Substituting the value of m_s, we obtain

$$N_i = \rho\, u_p^3 l^2/2.$$

The number of streams is found from the mass balance and is

$$i = V_{app}/V_{str} = \pi D^2 H/4l^3.$$

Substituting the values of i and N in (4), we obtain

$$\Sigma N = (\pi/8)\,\rho\, u_p^3 D^2 H/l. \quad (5)$$

Equating Eqs. (5) and (3), we find the velocity u_p from the known parameters and the experimental coefficient

$$u_p = \sqrt[3]{\frac{8\xi_м}{\pi} n^3 d_м^5 \frac{l}{D^2 H}}.$$

Substituting the value of n from the expression

$$n = u_c/\pi d_м,$$

we finally obtain

$$u_p = u_c \sqrt[3]{8\xi_м/\pi^4}\,(d_м/D)^{2/3}(l/H)^{1/3}.$$

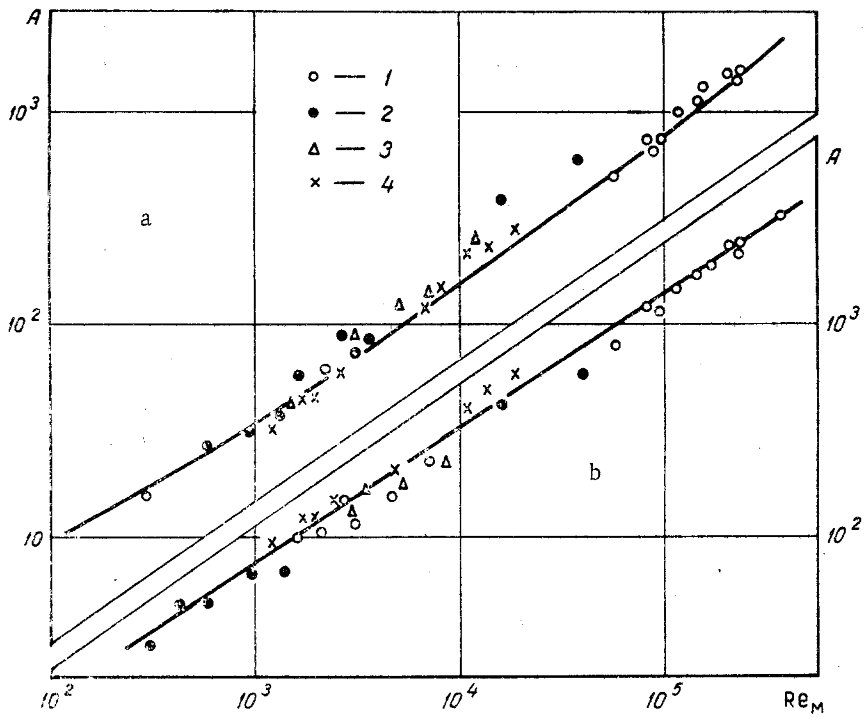

Fig. 1. Comparison of experimental data of [8] with values calculated from formulas (6) and (10) for paddle mixers $\left(A = \frac{\alpha D}{\lambda}\left(\frac{\mu_w}{\mu}\right)^{0,14} \mathrm{Pr}^{-1/3}\right)$: a) heat transfer at wall of apparatus; b) heat transfer at coil; 1) oil (heating through wall of apparatus, cooling by coil); 2) 92% solution of glycerol in water; 3) water; 4) oil (cooling through wall of apparatus, heating by coil).

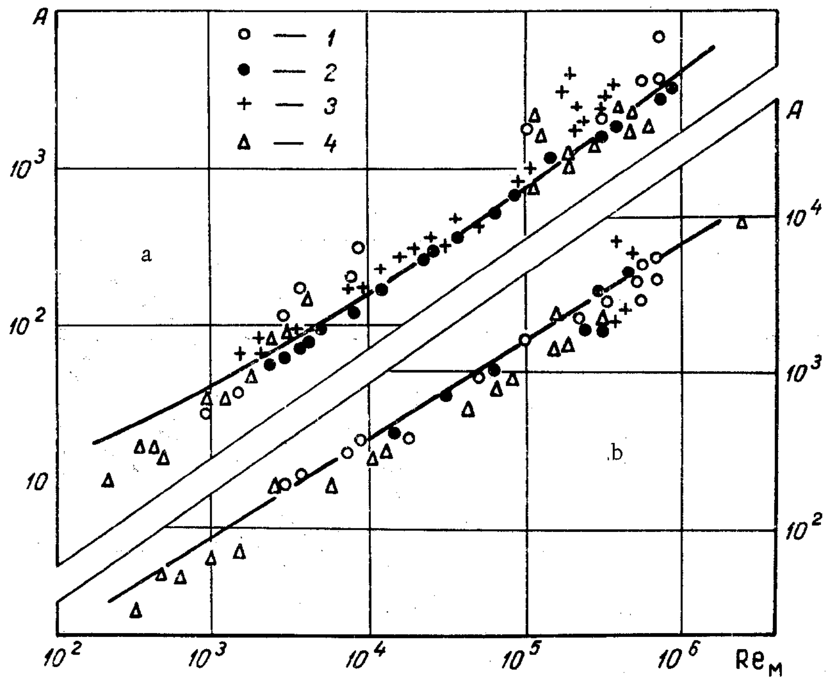

Fig. 2. Comparison of experimental data with calculation from (6) and (10) for turbine mixers $\left(A = \frac{\alpha D}{\lambda}\left(\frac{\mu_W}{\mu}\right)^{0.14} Pr^{-1/3}\right)$: a) heat transfer at wall of apparatus; heat transfer at coil; 1) data of [10]; 2) of [14]; 3) of [12]; 4) of [8].

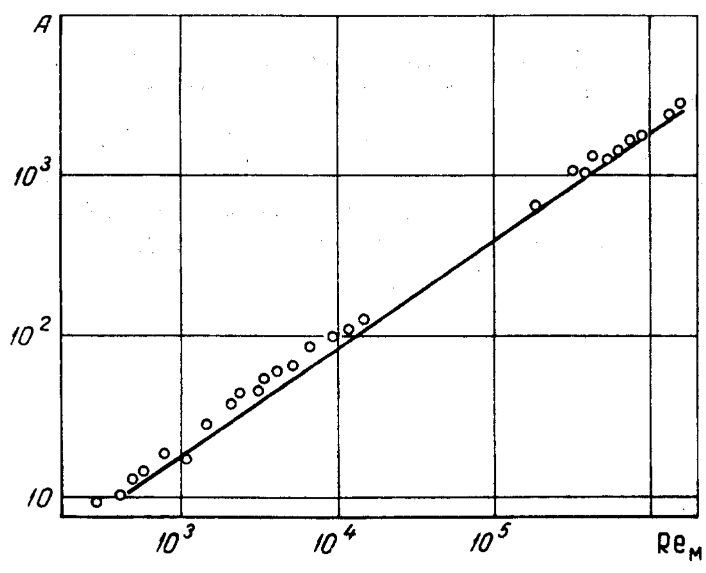

Fig. 3. Comparison of experimental data (points) of [13] with calculation (straight line) from formula (9) for heat exchange with a coil $\left(A = \frac{\alpha d_t}{\lambda}\left(\frac{\mu_W}{\mu}\right)^{0.14}\left(\frac{d_M}{D}\right)^{-0.1}\left(\frac{d_t}{D}\right)^{-0.5} Pr^{-1/3}\right)$.

Substituting now the found values of u_p and l in expressions (1) and (2), we obtain the formula for the heat transfer coefficient:

$$\frac{\alpha l}{\lambda}\mathrm{Pr}^{-1/3}\left(\frac{\mu_w}{\mu}\right)^{0.14}=$$

$$=\frac{1}{3}\,1.15\left[\frac{u_c}{}\frac{l}{\nu}\sqrt[3]{\frac{8\xi_M}{\pi^4}}\left(\frac{d_M}{D}\right)^{2/3}\left(\frac{l}{H}\right)^{1/3}\right]^{0.5}+$$

$$+\frac{2}{3}\,0.035\left[\left(\frac{u_c\,l}{\nu}\right)\left(\frac{8\xi_M}{\pi^4}\right)^{1/3}\left(\frac{d_M}{D}\right)^{2/3}\left(\frac{l}{H}\right)^{1/3}\right]^{0.8}.$$

Bringing the Re_M to the form used in the case of mixing

$$\mathrm{Re}_M = nd_p^2/\nu = u_c d_M/\pi\nu$$

and using the diameter of the vessel as the characteristic dimension for heat transfer, we finally obtain, after simple transformations,

$$\frac{\alpha D}{\lambda}\mathrm{Pr}^{-1/3}\left(\frac{\mu_w}{\mu}\right)^{0.14}=0.425\,\xi_M^{0.16}\,\mathrm{Re}_M^{0.5}\left(\frac{D}{d_M}\right)^{1/6}\left(\frac{D}{H}\right)^{1/6}\left(\frac{D}{l}\right)^{1/3}+$$

$$+0.032\,\xi_M^{0.2}\cdot 0.26\,\mathrm{Re}_M^{0.8}\left(\frac{D}{d_M}\right)^{0.25}\left(\frac{D}{H}\right)^{0.25}\left(\frac{l}{D}\right)^{0.065}. \quad (6)$$

Since the type of mixture was not taken into account in the derivation of this equation, the latter applies to any mixer where the distance between the blades and the wall is much greater than the thickness of the viscous sublayer.

To determine the heat transfer coefficient we need to know the diameter d_M of the mixer, the diameter D of the apparatus, and the power factor ξ_M.

For practical calculations expression (6) can be simplified and put in the following form:
for $30 < \mathrm{Re}_M < 1000$

$$(\alpha D/\lambda)\,\mathrm{Pr}^{-1/3}(\mu_w/\mu)^{0.14}=0.60\,\mathrm{Re}_M^{0.6}\,\xi_M^{0.2}(D/d_M)^{0.2}(D/H)^{0.25}; \quad (7)$$

for $\mathrm{Re}_M > 10^3$

$$(\alpha D/\lambda)\,\mathrm{Pr}^{-1/3}(\mu_w/\mu)^{0.14}=0.3\,\mathrm{Re}_M^{0.1}\,\xi_M^{0.2}(D/d_M)^{0.2}(D/H)^{0.25}. \quad (8)$$

Heat transfer to coils. Coils are often used to intensify heat transfer in apparatus with a mixer. As before, we can assume that the coil is washed by a quasi-steady stream of liquid with velocity up. The direction of flow over the coil varies from 90 to 0° because of the random nature of the turbulent pulsations.

For tubes washed transversely by the liquid we can obtain from the results of numerous investigations [2, 4]

$$\mathrm{Nu}_T=\alpha d_T/\lambda=0.17\,(u_p d_T/\nu)^{0.62}\,\mathrm{Pr}^{1/3}(\mu/\mu_w)^{0.14}\varepsilon_\varphi.$$

The characteristic dimension will be the diameter d_T of the tubes of the coil, since the length of the turbulent pulsations in this case is greater than the coil diameter.

Assuming that all directions of turbulent pulsations are equally probable, we find the mean value of the correction for the angle of attack ε_φ [4] by graphic integration of ε_φ and subsequent averaging

$$\varepsilon_\varphi=\frac{2}{\pi}\int_0^{\pi/2}\varepsilon_\varphi\,d\varphi=0.76.$$

Bringing $\mathrm{Re}_T = ud_w/\nu$ to the form used in the case of mixing we obtain

$$(\alpha d_T/\lambda)\,\mathrm{Pr}^{-1/3}(\mu_W/\mu)^{0.14}=$$

$$=0.135\,\mathrm{Re}_M^{0.62}\,\xi_M^{0.2}(d_T/D)^{0.38}(D/H)^{0.2}. \quad (9)$$

In some investigations the diameter of the vessel is taken as the characteristic dimension. In this case, transforming expression (9), we will have

$$(\alpha D/\lambda)\,\mathrm{Pr}^{-1/3}(\mu_w/\mu)^{0.14}\approx$$

$$\approx 0.13\,\mathrm{Re}_M^{0.62}\,\xi_M^{0.2}(D/d_T)^{0.62}(D/H)^{0.2}. \quad (10)$$

Expressions (9) and (10) can be used for an approximate calculation of the heat transfer to the surface of bodies of various configurations, since Shchitnikov's investigations [5] showed that heat transfer to bodies of different configuration can be satisfactorily described by a single relationship.

We will compare the obtained theoretical formulas (7)–(10) with the experimental heat-transfer data for apparatus with mixers of different types. To do this we need to know the power coefficient ξ_M, which was calculated by Kafarov's method [6] with due regard to the dimensions of the apparatus near the mixer.

Paddle mixers. Uhl [7] and Chilton [8] investigated heat transfer to the wall of an apparatus with a paddle mixer. Uhl conducted experiments in a smooth-walled vessel without a coil, while Chilton investigated a vessel with a coil. As Fig. 1 shows, the agreement between the experimental and theoretical data is quite satisfactory in view of the approximate nature of the formulas. The disagreement is 10–20% and is practically within the limits of experimental error.

Propeller mixer. From experimental data for a propeller mixer without a coil Ackley [1] obtained the following equation:

$$(\alpha D/\lambda)\,\mathrm{Pr}^{-1/3}(\mu_w/\mu)^{0.14}=0.36\,\mathrm{Re}_M^{0.67}.$$

Determining the value of ξ_M and substituting the value (D/d_M) for a propeller mixer in Eq. (13), we obtain

$$(\alpha D/\lambda)\,\mathrm{Pr}^{-1/3}(\mu_w/\mu)^{0.14}=0.3\,\mathrm{Re}_M^{0.67},$$

which is fairly close to the experimental result.

Horseshoe mixers. Uhl [7] investigated heat transfer in vessels with horseshoe mixers. Uhl's data are described by the following expression:

$$(\alpha D/\lambda)\,\mathrm{Pr}^{-1/3}(\mu_w/\mu)^{0.14}=c\,\mathrm{Re}_M^n, \quad (11)$$

where

$$\text{for } \mathrm{Re}_{\mathrm{M}} \leqslant 300 \quad c = 1.0, \ n = 0.5;$$
$$\text{for } \mathrm{Re}_{\mathrm{M}} \geqslant 300 - 4000 \ c = 0.38, \ n = 0.67.$$

The theoretical data are also approximated by Eq. (11) with the following values of constant and index:

$$\text{for } \mathrm{Re}_{\mathrm{M}} < 300 \ c = 0.85, \ n = 0.5,$$
$$\text{for } \mathrm{Re}_{\mathrm{M}} > 300 \ c = 0.325, \ n = 0.67.$$

The agreement between the experimental data and the calculated data is quite satisfactory.

Turbine mixers. Heat transfer in apparatus with turbine mixers ([9–1], etc.) has been most fully investigated.

Figure 2 shows that the agreement between the experimental and theoretical data can also be regarded as satisfactory.

Most of the experimental data for turbine mixers in the case of well-developed turbulence ($\mathrm{Re}_M > 1000$) are satisfactorily described by relationship (8) with $c = 0.4$ and $n = 0.67$.

The theoretical values of c and n are

$$c = 0.38, \ n = 0.67.$$

Several experiments on turbine mixers have been conducted in apparatus with baffles and coils. According to the results, the introduction of baffles increases the heat transfer coefficient by approximately 30–50%.

It follows from expression (8) that

$$\alpha \sim \xi_{\mathrm{M}}^{0.2}.$$

Assuming that at high values of Re_M the introduction of baffles increases the power coefficient by a factor of 4–10, we find that the heat transfer coefficient should be increased by a factor of 1.3–1.5, which agrees with the experimental data.

The introduction of a coil [12] increases the heat transfer coefficient to the wall of the apparatus by 20–30%. This is accompanied by an increase in the power coefficient ξ_M by a factor of 2–4. Heat transfer to coils (tubes) inside the apparatus has been investigated by several research workers [8, 10, 12–14]. A comparison of the experimental data of [8] and [10] with the data calculated from Eq. (10), where the diameter of the apparatus was chosen as the characteristic dimension, is given in Figs. 1 and 2.

These figures show that the agreement between the experimental and theoretical data is quite satisfactory.

A comparison of the experimental data of [13] with the theoretical data obtained with the diameter of the coil tubes taken as the characteristic dimension (9) showed that in this case also the experimental and theoretical data were close (Fig. 3). There is good agreement not only as regards the constants and index for the Re_M, but also as regards the index for the ratio (d_T/D) (experimental 0.5; theoretical 0.38).

A comparison of the empirical heat transfer equations for different types of mixers shows that these equations are almost identical. This agrees with the formulas obtained by calculation and confirms the main idea of this work—the local similarity of the turbulent structure of the flow in apparatus with mixers of different design.

Thus, despite the rough assumptions, the obtained results account quite well for the known experimental data and allow an additional consideration of the dimensions of the mixer and apparatus. This justifies a similar approach to other complex cases.

Although the turbulent flow is incomparably more complicated and the turbulence in the considered case is nonuniform, the effect of this nonuniformity on the integrated characteristics (heat transfer coefficient, etc.) is slight. This is exemplified by the choice of the characteristic dimension of the pulsations.

An analysis of the formulas indicates that $\alpha \sim l^n$, where $n \approx 0.15$–0 (n tends to zero at high Reynolds numbers).

A similar conclusion can be reached in regard to the effect of unsteadiness. It is shown in [15] and [16] that in the case of low-frequency pulsations the unsteady nature of the flow has practically no effect on the rate of heat transfer.

NOTATION:

D—diameter of apparatus; d_M—diameter of mixer; H—height of apparatus; d_T—diameter of tubes of coil; l—characteristic dimension of turbulent pulsations; n—number of revolutions of mixer; u_c—circular velocity of end of mixer; u_p—averaged fluctuation velocity; N—power consumption of mixer; N_i—power of elementary stream of liquid; m_s—mass flow through section l_T^2, per second; α_n—heat transfer coefficient for pulsations normal to wall; α_t—heat transfer coefficient for pulsations tangential to wall; $\bar{\alpha}$—mean heat transfer coefficient; λ—thermal conductivity; ν—kinematic viscosity; ρ—density of liquid; μ—dynamic viscosity; ξ_M—power factor; Re_M—centrifugal Reynolds number; Re—pulsation Reynolds number; Nu—Nusselt number, Pr—Prandtl number.

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